Marginal Expected Shortfall and Related Systemic Risk Measures
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ERFIN Workshop, 16th September, 2016

A very up-to-date and interesting topic on the validation of systemic risk measures.

$Y_t = (Y_{1,t}, Y_{2,t})$ – vector of stock returns for two assets at time t.

The **Marginal Expected Loss (MES)** is defined as (see Acharya et al. 2010, Brownlees and Engle 2015):

$$MES_{1t}(\alpha) = \mathbb{E}(Y_{1t} | Y_{2t} \leq VaR_{2t}; \Omega_{t-1}) \tag{1}$$

whereas the **Conditional Value at Risk (CoVaR)** is:

$$CoVaR_{1t} = F^{-1}_{Y_1|Y_2 \leq VaR_2 t(\alpha)}(\beta; \Omega_{t-1}) \tag{2}$$

Backtesting risk measures is based on the joint violation process:

$$H_t(\alpha;\theta_0,\Omega_{t-1}) = (1 - u_{12t}(\theta_0))\mathbf{1}(u_{2t}(\theta_0) \leq \alpha) \tag{3}$$

where

$$\begin{aligned} u_{12t}(\theta_0) &= u_{12t} = \int_0^{Y_1} f_{Y_1|Y_2 \leq VaR_2 t(\alpha)}(u;\Omega_{t-1},\theta_0)\,\mathrm{d}u \\ &= F_{Y_1|Y_2 \leq VaR_2 t(\alpha)}(Y_{1t};\Omega_{t-1},\theta_0) \end{aligned} \tag{4}$$

$$u_{2t}(\theta_0) = u_{2t} = \int_0^{Y_2} f_{Y_2}(u;\Omega_{t-1},\theta_0)\,\mathrm{d}u = F_{Y_2}(Y_{2t};\Omega_{t-1},\theta_0) \tag{5}$$

We know that $u_{12t} \sim i.i.d.\ U(0,1)$ and $u_{2t} \sim i.i.d.\ U(0,1)$.
Hence,

$$\mathbb{E}(H_t(\alpha,\theta_0) = \frac{\alpha}{2} \tag{6}$$

Two major testing procedures proposed in the paper are:

(1) Unconditional Coverage test, with

$H_{0,UC}: \mathbb{E}[H_t(\alpha, \theta_0)] = \frac{\alpha}{2}$

(2) Test of "independence" , where:

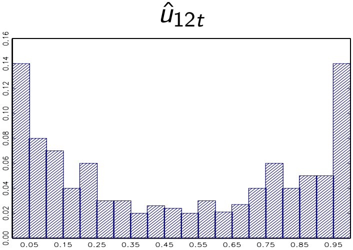
$H_{0,Ind}: \rho_1 = \rho_2 = \ldots = \rho_m = 0.$

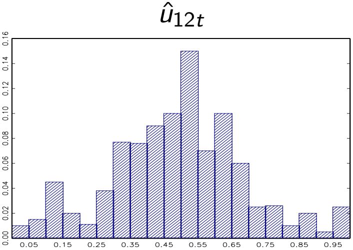
The underlying data is:

$$\hat{H}_t(\alpha; \hat{\theta}_0, \Omega_{t-1}) = (1 - \hat{u}_{12t})\mathbf{1}(\hat{u}_{2t} \leq \alpha) \quad (7)$$

Are the tests sensitive enough to account for various forms of model misspecification?

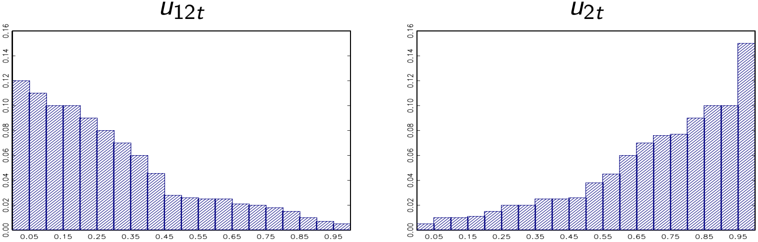
(1) What happens if the conditional density function $f(Y_1|Y_2 \leq VaR_{2t}, \Omega_{t-1})$ is misspecified although $f(Y_2|\Omega_{t-1})$ is correctly specified?





If the tails of Y_2 distribution (or/and dependence between two marginals) are misspecified, then $\hat{u}_{12}$ distribution could heavily diverge from uniformity (although $\hat{E}(1 - \hat{u}_{12})$ can still be very close to 0.5). Will the test reject the null?

(2) What happens if the two sources of misspecification, i.e. $f(Y_1|Y_2 \leq VaR_{2t}, \Omega_{t-1})$ and $f(Y_2|\Omega_{t-1})$ cancel out?



Let's assume that (1) $\alpha-$quantile of true DGP for Y_2 is underestimated (Then an event $\hat{u}_{12,t} \leq \alpha$ can occur (let's say) $\frac{5}{8}\alpha$ times, and (2) $\hat{E}(1 - \hat{u}_{12})$ will be bigger then $\frac{1}{2}$, say $\frac{8}{10}$. Then the UC test statistic will be equal to 0 suggesting perfect goodness-of-fit.

(3) Shouldn't the Independence Test account for the dependence in higher moments? ACF of $(\hat{u}_{12} - \bar{u}_{12})^2$ and $(\hat{u}_{12} - \bar{u}_{12})^3$ can be helpful. Visual presentation of PIT has been advocated by Diebold, Gunther and Tay (1998) in the *International Economic Review* and Diebold, Hahn and Tay (1999) in the *Review of Economics and Statistics*.

Diverse applications of PIT in validation of econometric models include papers of Bauwens et al. (2001) in *International Journal of Forecasting*, Hautsch (2003, 2013) in the *Journal of Financial Econometrics*, Liesenfeld et al. (2006) in the *Empirical Economics*, Genest et al. (2006) in the *Scandinavian Journal of Statistics*.